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On the Value of the Long Inequality in the Motion of the Moon due to the Disturbing Action of the Planet Mars. By Edmund Neison.

In the *Monthly Notices* for November (vol. xlv. p. 1) there is a communication by M. Gogou on the difference between the values found by us for the coefficient of the long inequality in the motion of the Moon with the argument: the mean motion of the Moon plus twenty times the mean longitude of *Mars* minus twenty-four times the mean longitude of the Earth.

The difference between M. Gogou and myself may be briefly stated thus. Whilst we both agree that the true value of the coefficient of this term when found *in this way* is only a small fraction of a second of arc, whereas M. Gogou's computations lead to values amounting to less than one thousandths of a second of arc, mine lead to values nearly a thousand times greater—to almost as large a quantity as that found in a similar way by M. Delaunay for the coefficient of the term of similar character due to the action of *Venus*, and having the argument: eight times the mean longitude of *Venus* minus thirteen times the mean longitude of the Earth.

There is more attached to this difference, *as thus stated*, than is thought by M. Gogou, who assumes it to be immaterial, as may be seen by an examination of the origin of the similar discordance between the values obtained by Hansen and Delaunay for the coefficient of the *Venus* term already mentioned.

As M. Gogou's calculations have been carried out with so much care, skill, and elaborateness, I should have been disposed to look for the source of this remaining discordance in some unsuspected deficiencies in my own far less elaborate work. But an examination of his memoir indicated a more probable origin. It was this. In calculating the values of the coefficients denoted by B, I retained the terms depending on the fifth and seventh powers of the eccentricities of *Mars* and the Earth, but in his calculation M. Gogou has omitted to do this. This I pointed out in my Note in the *Monthly Notices* for March 1882 (p. 266), and it seemed to me to offer a most probable explanation of the difference between my revised results and those of M. Gogou. I say revised results, for I had already in the *Monthly Notices* for March 1878, or four years earlier, pointed out that the very large value given in my first communication, in November 1877, was far too great, as the ordinary methods of calculating the coefficients b^i which I had employed were not sufficiently accurate for this special term.

In his present communication, M. Gogou returns to the consideration of this point and calculates the effect of certain small subsidiary terms of the fifth order, which were included by me in my calculations, but were omitted by him from his own investigations. He shows that the omission of these terms fails to account for the existing discordance.

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But this result is really beside the question, for as a reference to my Note will show, it is clearly not to these terms of the fifth order to which I allude in my remarks. M. Gogou has failed to see the point of my criticism, as, indeed, my reference to terms of the seventh order might have shown him had he not disregarded it.

I will, therefore, now make my meaning unmistakable.

When we expand Δ^{-s}, or the s^{th} power of the reciprocal of the geocentric distance of *Mars* in the form of the series

$$(a''')^{-s}\Sigma\left[\mathrm{B}_{\frac{s}{2}}\right]^{i}_{i+k}\cos\{[in''-(i+k)n''']t+\mathrm{E}\},$$

the coefficients have the form

$$\left[\mathrm{B}_{\frac{s}{2}}\right]^{i}_{i+k}=(e''')^{k}(e'')^{i-k}\{\mathrm{N}_0+\mathrm{N}_1(e''')^2+\mathrm{N}'_1(e'')^2+\mathrm{N}_2(e''')^4$$
$$+\mathrm{N}'_2(e''')^2(e'')^2+\mathrm{N}''_2(e'')^4+\ \ldots\}$$

where N_0, N_1, N_1' . . . are numerical coefficients.

In the ordinary development of the planetary theory, the terms involving N_1, N'_1, &c., are very much smaller than the first term depending on N_0, and can be neglected, therefore, in nearly every case. But in the present case this is by no means true, for owing to the large value of i, and the magnitude of both a and e''', the terms depending on N_1 and N_2 are nearly as large as that depending on N_0. They must therefore be retained. Now, this is what I have done, but it is what M. Gogou has omitted to do.

It must not be supposed that this omission is necessarily unimportant, because as the terms depending on N_0 are so small, even if those depending on N_1 were as large, their sum would still be unimportant. The results depending on $\mathrm{N}_0{}^k$ are small, putting $\mathrm{N}_0{}^k$ to denote the value of N_0 in the coefficient

$$\left[\mathrm{B}_{\frac{s}{2}}\right]^{i}_{i+k},$$

because of the mutual relations between them. But as the same relations do not exist amongst $\mathrm{N}_1{}^k$ as amongst $\mathrm{N}_0{}^k$, it is easy to see that the mutual relations which destroy the one will not destroy the other. Hence the terms in N_1 ought to yield far larger results than the terms in N_0.

It is to this difference in our investigations that I ascribe the existing discordance in our results, I having retained many more terms in my calculations than have been included by M. Gogou in his more detailed and elaborate work.

I must add that having neither M. Gogou's memoir nor my own calculations of this term with me in Natal, I am quite unable until my return to England to separate these different portions of my work, and so clearly prove the accuracy of this view of the origin of the discordance between our results.

The complete value of the coefficient of this term practically depends on two functions—one arising from the direct action of *Mars* on the motion of the Moon as disturbed by the action of the Sun; and the other depending on the direct action of the

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Sun on the perturbations in the motion of the Moon produced by the action of the planet.

In the preceding remarks I have confined myself to the former of these two portions, as it is the sole one which has been dealt with by M. Gogou, who, like Delaunay, has overlooked entirely the second more important portion. It is this second portion which forms the only real difficulty in the accurate calculation of the terms of this kind. It is the omission of this second portion by Delaunay which explains the discordance between him and Hansen with regard to the values of the long inequalities due to *Venus*, and it is the calculation of this second portion which seems to have so completely baffled Hansen in his attempt to deal with it in a numerical form.

But until this additional portion has been properly computed it cannot be said that the value of this term has been shown to be insensible, as believed by M. Gogou.

I had hoped to have ere this finished and published a complete determination of the real values, not only of this term, but also of both Hansen's terms of long period; but being unable out here in Natal to obtain a copy of Pontécoulant's *Théorie de la Lune*, a number of whose developments I have used in my subsidiary calculation, I am forced to delay this until I have recalculated nearly the whole of these developments—probably nearly a year's additional work.

Natal Observatory:
1885, *Feb.* 3.

Observations of Barnard's Comet (*b* 1884) *made at the Natal Observatory, Durban.* By Edmund Neison.

In the telegram sent to this Observatory on July 22, the North Polar Distance of this comet was erroneously given as 120° instead of 127°; so, though looked for most carefully, it was not discovered before the night of August 11. It was a faint, rounded, nebulous mass, with a central condensation. Sketches were made of the appearances of the comet on the nights of August 12 and 21, and September 13 and 16. The following notes were also recorded.

August 12.—Round hazy body with a hazy nucleus, and a faint tail-like extension.

August 16.—Very faint. Rounded preceding edge and faint tail. No distinct nucleus.

August 18.—Comet faint nebulous mass, without definite edges or nucleus. Slight evidence of tail on following side.

August 21.—Comet very faint. About 2′ in diameter. Two tiny points of light in the centre resembling 15-magnitude stars, which they probably were.

September 13.—Comet about 2′ in diameter, and decidedly condensed towards the centre.